Homework #3

Due Feb 28, 2024, 11:59pm

Problem 3.1.

The files `noise01.csv` to `noise10.csv` contain a random noise from a real instrument, measuring the intensity of light as a function of the voltage on a light source. The voltage goes from 0.1V to 1.0V, encoded in the filename. (0.1V, 0.2V, 0.3V, 0.4V, 0.5V, 1.0V). Prove that the noise is due to the Poisson distribution of the discrete photons using iPython. Hint: Use the fact that a Poisson distribution has a single parameter, which determines both its mean and variance. Show that these quantities satisfy the appropriate scaling law for each data set.

Problem 3.2.

We are drawing N random variates from an exponential distribution:

$$p(x) = e^{-x}$$

Work out analytically what is the expected probability distribution of the maximum of N samples. Also, work out the analytic distribution of the second largest element. Write the derivation into the notebook. Run numerical experiments with N=100,5000,20000, repeat each run 100 times, and show how the empirical results compare to the predictions.

Problem 3.3.

We have a person taking a step with a length of 1 in a random direction in the 2D plane. Build a numerical simulation of this process, and save the distance traveled after 10, 100, 1000, and 10000 steps, 4 outputs per simulation. Then run the simulation 1000 times, and determine the expectation value of the distance travelled as a function of the number of steps. Also, plot the probability distribution of the distance travelled at each “snapshot”.

Problem 3.4.

This problem is based on an old TV show called “Let’s Make a Deal”. The stage of that show had 3 doors numbered “1”, “2”, and “3”. Behind one of the doors was a valuable prize, the other two contained weird, smaller gifts. The contestant would choose one of the doors, say “2”, but it would not be opened yet. The host would then open one of the *other* doors that *always* had a gag gift behind it (say, door “1” for our example). He would then ask the contestant if he or she wanted to stay with door “2”, or change their selection to door “3”. The problem is to determine which action – keep or change – gives the contestant the greater probability of selecting the door with the real prize. The task is to write a simulation code that will play a large number of games on the computer, always switching doors (or never switching doors), and record whether we won. We then calculate the respective probabilities of a win for both strategies.