Homework #6

Due March 7, 2022, 11:59pm

Problem 6.1.

Read the `sidneybw1000.png` gray-scale image, 1000x750 pixels, dtype='uint8'. Convert the image to a 2D numpy array. Then, compute the first 200 eigenvalues and eigenvectors.

(i) Plot the spectrum of the eigenvalues as a function of their rank.

(ii) Also plot the cumulative fraction of the variance in each mode.

(iii) Finally, use only the N largest eigenvalues, reconstruct the truncated image, and display, for different values of N, from 10 to 100. Determine, which gives an adequate quality reconstruction of the image. For each value of N, compute the amount of storage needed and compare it to the original image size to get the compression ratio.

(iv) Repeat the exercise with the `einstein.png` image.

Hints:

(a) for reading the image use the `imageio` package.
(b) For a truncated SVD, use `scipy.sparse.linalg.svds`.
Beware that in svds the largest eigenvalues are last.
(c) For computing the dot products of two matrices A and B use the operator A @ B
(d) The eigenvalues from the image SVD need to be converted to represent the variance

Problem 6.2.

The `temperature.csv` text file contains the daily mean temperature in F^{o} for the cities Helsinki and Melbourne for the years 2013 and 2014. The first column is the day measured from 01-01-2013. Build a linear model that fits the temperature variations with a linear combination of sin and cos functions. The fundamental period should be 1 year (365 days), use up to the third harmonic. Plot the best fit solutions on top of the data.

Hint: Watch out for the header line in the text file