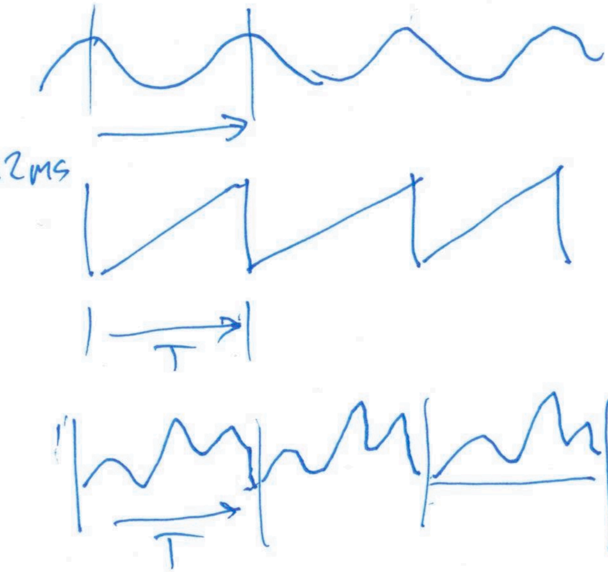
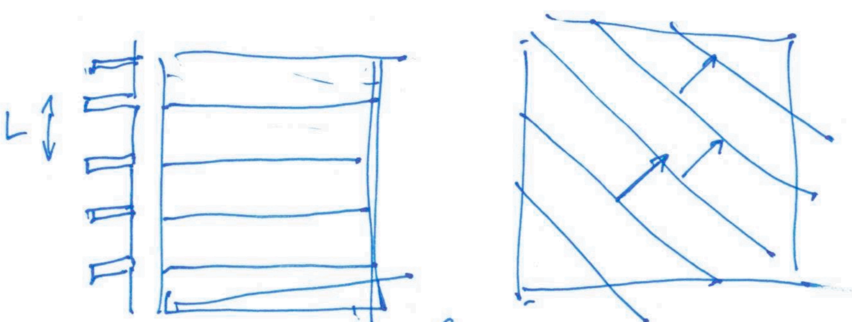
# Fourier transform

$f(t) = A \cdot \sin[2\pi\, 440\, t]$ 440 Hz sinusoidal

Periodic $f(t) = f(t+T)$

$T = \frac{1}{440\,\text{Hz}} = \frac{1}{440}\,s \sim 2.2\,ms$




spatial waves

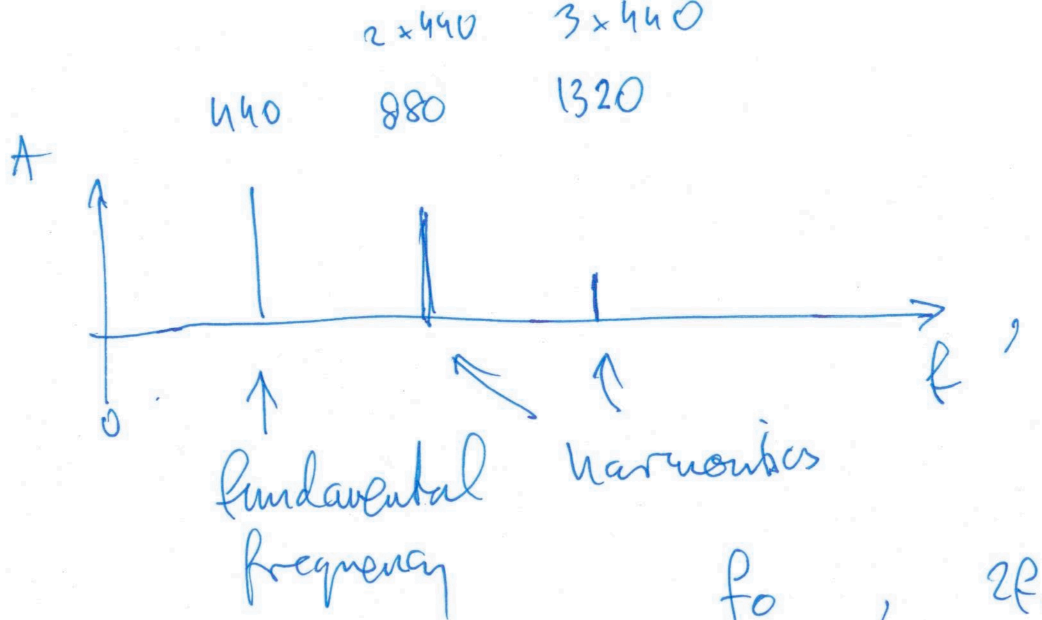
$$f(t) = f(t+T) = f(t+nT).$$

$$g(t) = A \cdot \sin 2\pi 440 t + B \sin 2\pi 880 t + C \sin 2\pi 13$$

$$nT = \frac{1}{880}$$

$$A \sin 2\pi\, 440 \left(\frac{n}{880}\right) + B \cdot \sin 2\pi 880 \left(\frac{n}{880}\right) \qquad n = 1$$

$$A \sin 2\pi \left(\frac{n}{2}\right) + \underline{B \cdot \sin 2\pi \cdot \cancel{88}\, n}$$




Frequency spectrum

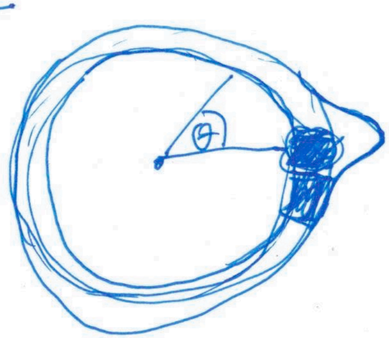
$f_0 \ , \ 2f_0 \ , \ 3f_0 \ , \ 4f_0 \ , \ \ldots$

$$A \sin 2\pi f_0 t + \ldots + C \sin 2\pi (k f_0) \cdot t \qquad T = \frac{1}{f_0}$$

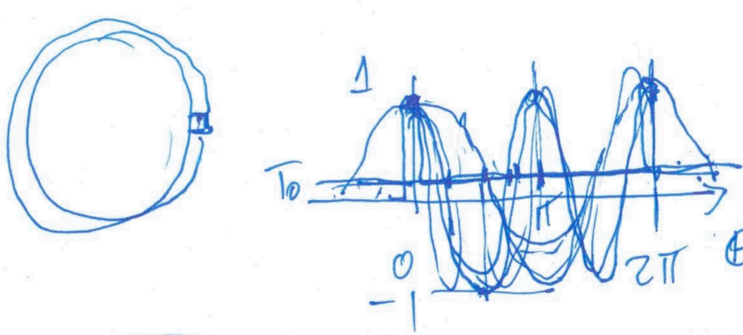
$$\underline{A \sin 2\pi \cancel{f_0} \frac{1}{\cancel{f_0}}} \quad \ldots \quad C \sin(2\pi k \cancel{f_0}) \frac{1}{\cancel{f_0}}$$

$$\underline{\sin 2\pi k}$$

<u>Fourier</u>




$$T(\theta) = \sum_{n=0}^{\infty} A_n \cdot \underline{\sin(n\theta + \phi_n)}$$




$T_0 = A_0 \sin(\phi_0)$ zero freq

$A_1 \cos(\theta + \cancel{\phi_1})$

$A_2 \cos(2\theta + \cancel{\phi_2})$

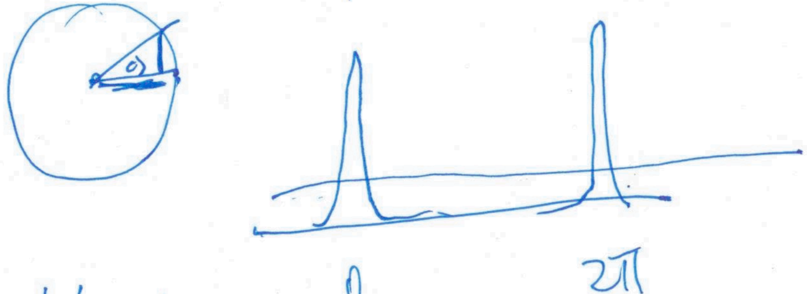
$A_3 \cos(3\theta + \cancel{\phi_3})$

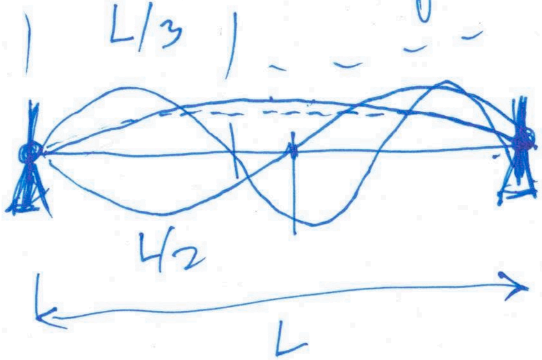
$\vdots$

$A_n$ .

$\cos 0 = 1$

$\sin 0 = 0$




Form a harmonic sequence

$C^\#$ $D^\#$ $F^\#$ $G^\#$ $A^\#$

C D E F G A B C ..... C

1:1 9:8 5:4 4:3 3:2 5:3 15:8 2:1 4:1

$f_0$

Bach Wohltemperierte Klavier

$2^{1/12}$

$e^{\left[\frac{1}{12} \ln 2\right]} = \left(\frac{C^\#}{C}\right) = \left(\frac{D}{C^\#}\right)$

Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + \sum b_n \sin(2\pi n f_0 t)$$

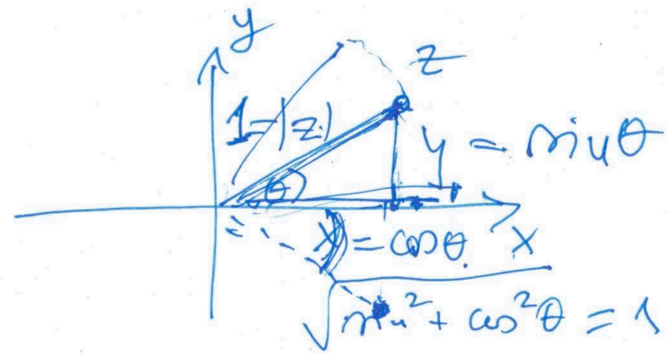
$$= \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \sin(\underbrace{2\pi n f_0 t}_{x} + \underbrace{\phi_n}_{y})$$

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \sin y = \underline{\sin(2\pi n f_0 t)} \cos\phi_n + \underline{\cos(2\pi n f_0 t)} \sin\phi$$

$$A_n^2 = a_n^2 + b_n^2 \dots$$

$$z = x + iy \qquad i = \sqrt{-1}$$
$$i^2 = -1$$

$\operatorname{Re} z$ $\quad$ $\operatorname{Im} z$




$$|z| = \sqrt{x^2 + y^2}$$

$$z^2 = (x+iy)(x+iy) = x^2 + 2ixy + i^2y^2$$

$$z^2 = x^2 - y^2 + 2i(xy) = \underbrace{\cos^2\theta - \sin^2\theta}_{\cos 2\theta} + i\,\underbrace{2\sin\theta\cos\theta}_{\sin 2\theta}$$

$$|z| = 1$$

$$z = x + iy = \cos\theta + i\sin\theta$$

$$(x+iy)(x-iy) = x^2 - i^2y^2 + ixy - ixy = x^2 - i^2y^2 = x^2 + y^2$$

$$\cos^2\theta + \sin^2\theta = 1$$

$z^* = x - iy$ complex conjugate

$z = \underline{x + iy}$

$z \to (x, y) \equiv (r, \theta)$ $\quad r^2 = x^2 + y^2$ $\quad x = r\cos\theta$

$y = r\sin\theta$

$|z| = 1$ $\quad z = \cos\theta + i\sin\theta = e^{i\theta}$

$z^2 = \underline{\cos 2\theta} + i\,\underline{\sin 2\theta} = e^{i2\theta} = e^{i\theta} \cdot e^{i\theta} = e^{i(\theta+\theta)} = e^{i2\theta}$

$z^3 = e^{i3\theta} = \cos 3\theta + i\sin 3\theta$

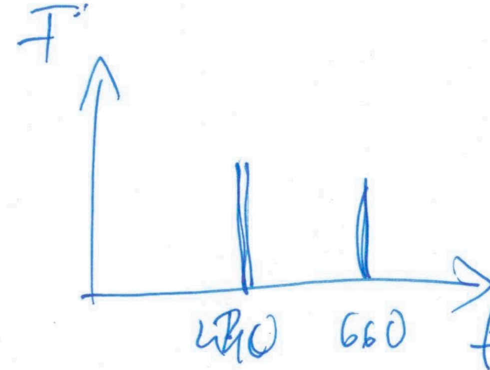
$f(t) = \sum a_n \cdot e^{i2\pi n f_0 t}$ $\quad 2\pi f_0 = \omega_0$

$\sum a_n e^{in\omega_0 t}$

Writing periodic signals in complex form is MUCH SIMPLER!

Fourier transform

$$\hat{f}(k) \int_0^{\infty} e^{-2\pi i k x} f(x)\,dx$$



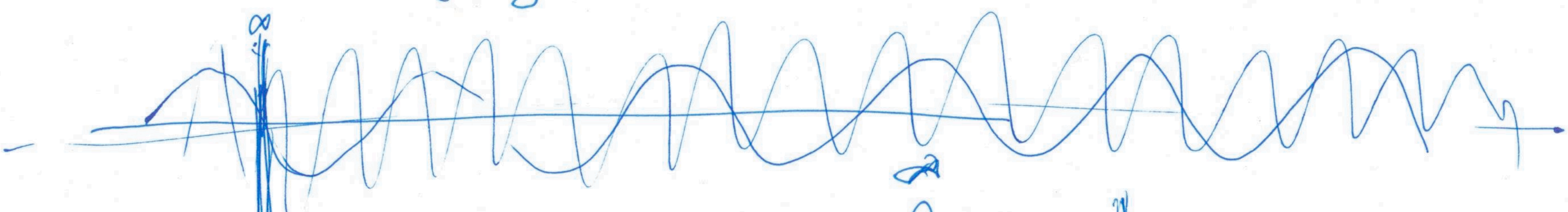

$$f(t) = A.\sin(2\pi\, 440\, t) + B.\sin(2\pi\, 660\, t)$$

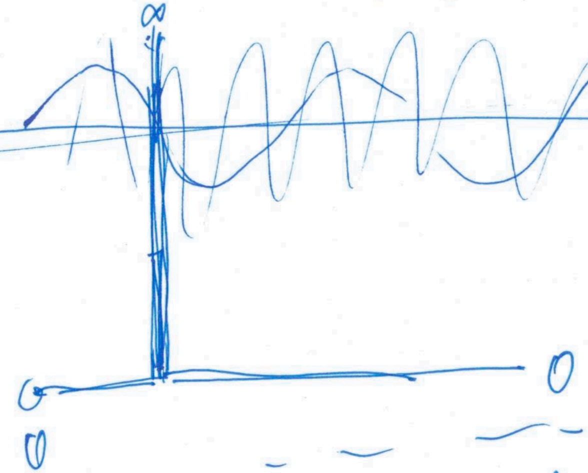
Getting the frequency spectrum.

$$F(f) = \int_{-\infty}^{\infty} f(t)\cdot \sin(2\pi f t)\,dt = \int_{-\infty}^{\infty} \sin(2\pi f t)\sin(2\pi g t)\,dt = \begin{cases} 0 & \text{if } g \neq \\ \infty & \end{cases}$$

$\sin(2\pi g t)$

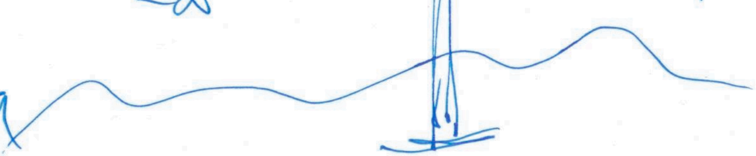
$\delta(g-f)$



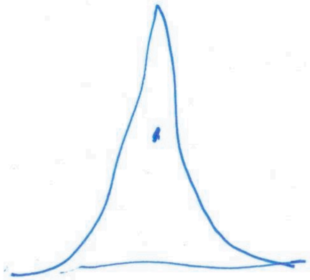



$$\int_{-\infty}^{\infty} \delta(x-y)\, f(y)\,dy = f(x)$$

DiRAC delta

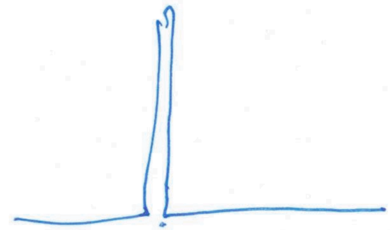


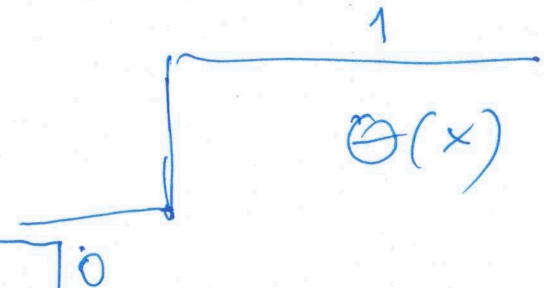
$\nu = \sigma^2$



$$\int_{-\infty}^{\infty} \frac{e^{-\frac{x^2}{2\nu}}}{\sqrt{2\pi\nu}}\, dx = 1$$

$$\frac{e^{-\frac{x^2}{2\nu}}}{\sqrt{2\pi}} \longrightarrow \delta(x)$$






$\Theta(x)$

DISTRIBUTIONS

$$\boxed{2\pi\,\delta(x-y) = \int_{-\infty}^{\infty} \sin 2\pi x t \,\sin 2\pi y t \, dt}$$

Discrete Fourier Transform