BLIND SOURCE SEPARATION

"Coctail - party problem"

$Z \in \mathbb{R}^{m \times n}$ matrix of many independent sources (n), with m samples

Signals mixed together using an $n \times n$ matrix A

$$X^T = A . Z^T$$

We are trying to unmix the observations by finding $W \in \mathbb{R}^{n \times n}$

$$Y^T = W X^T \quad \text{so that} \quad Y \approx Z.$$

1) Mixture is linear
2) Sources are independent
3) Mixture is stationary (constant)
4) The number of observations is the same as the # of sources

$$X = U\Sigma V^T \Rightarrow X^T = V\Sigma^T U^T \Rightarrow U^T = \Sigma^{-T} V^T X^T$$

each column is independant/orthogonal

$$\underbrace{W = \Sigma^{-T} V^T}$$

More general: ICA : Independent Component Analysis

One additional criterion: 1) Signals independent

2) Each component has a non-Gaussian distribution.

Steps: 1) Perform a PCA.

2) Do a whitening (rescale each axis by the sqrt(λ).)

3) Find the directions of maximum non-Gaussianity

- Kurtosis
- Projection pursuit

ICA components are not orthogonal

$$K = \frac{E[(y-\bar{y})^4]}{E[(y-\bar{y})^2]^2} - 3$$

Linear Discriminant Analysis LDA

Find a linear combination of features that separates 2 or more classes $\Rightarrow$ Linear classifier

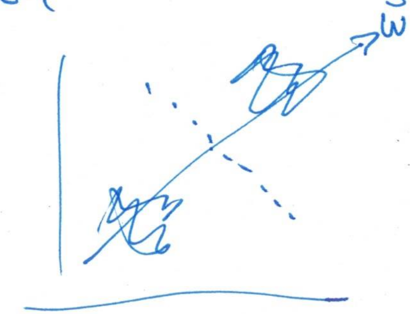
Two classes $\vec{\mu}_0, \vec{\mu}_1$ Σ_0, Σ_1

Probability

$$\frac{\exp\left(-\frac{1}{2}\cdot(\vec{x}-\vec{\mu})^T\Sigma^{-1}(\vec{x}-\vec{\mu})\right)}{\sqrt{(2\pi)^n\det\Sigma}} = P(\vec{x})$$

The projection onto $\vec{w}$ $\Rightarrow$ $\mu_i' = \vec{w}\,\vec{\mu}_i$; $Var(x_i') = \vec{w}^T\Sigma_i\vec{w}$

Which is the direction giving max separation



Typically we do PCA, truncate, and find LDA

PRINCIPAL COMPONENT PURSUIT

Decompose signal into low-rank (PCA) & sparse.

$$X = \underbrace{L}_{\text{low-rank}} + \underbrace{S}_{\text{sparse}} + D$$

Principal Component Pursuit

- Low rank approximation of data matrix: X
- Standard PCA:

$$\min \|X - E\|_2 \quad subject\ to\ \ rank(E) \le k$$

  - *works well if the noise distribution is Gaussian*
  - *outliers can cause bias*
- Principal component pursuit

$$\min \|A\|_0 \quad subject\,to\ \ X = N + A,\ rank(N) \le k$$

  - *"sparse" spiky noise/outliers: try to minimize the number of outliers while keeping the rank low*
  - *NP-hard problem*
- The L1 trick:

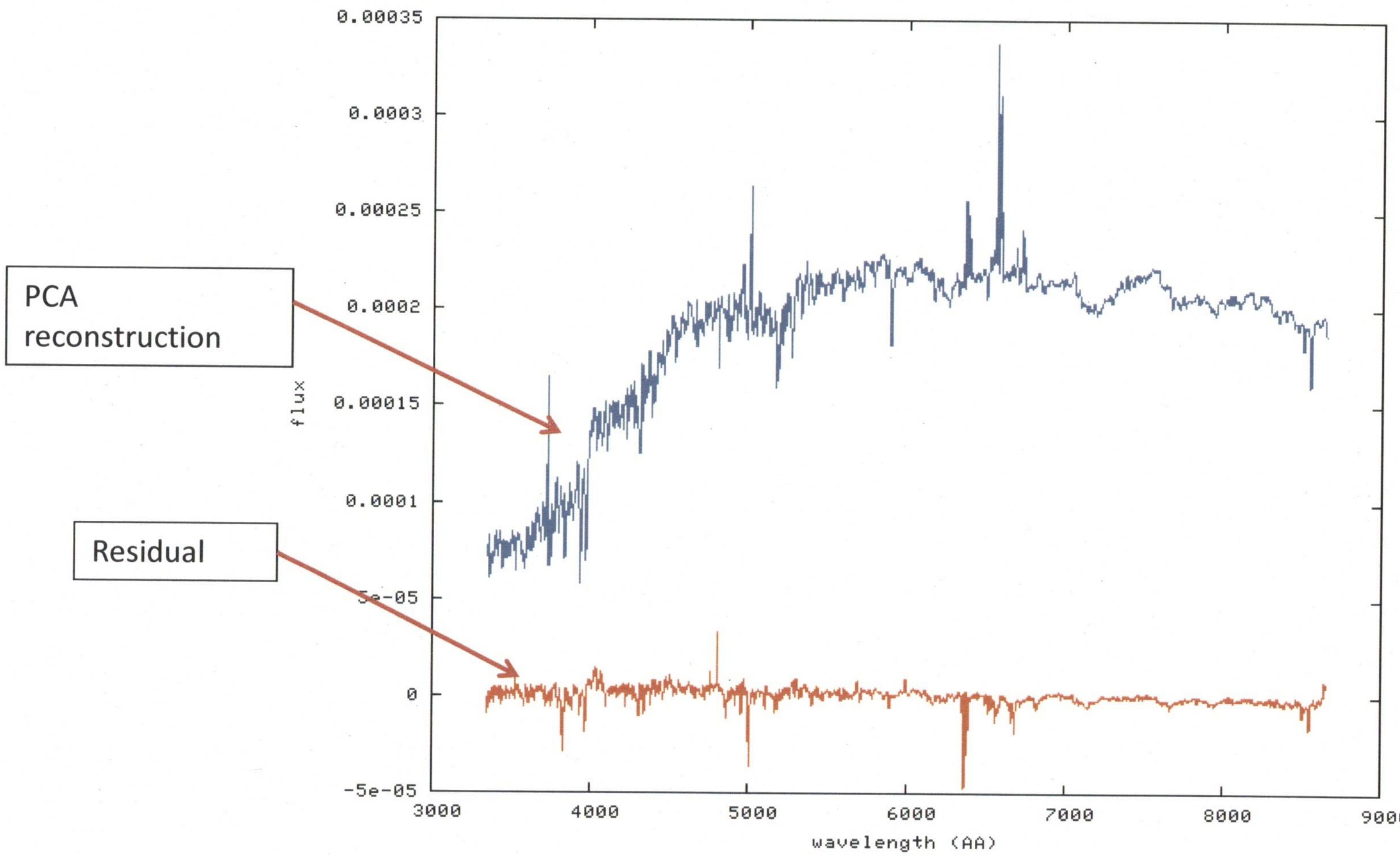
$$\min_{N,A}\left(\|N\|_* + \lambda\|A\|_1\right) \ subject\,to\ \ X = N + A$$

  - *numerically feasible convex problem (Augmented Lagrange Multiplier)*

$$\min_{N,A}\left(\|N\|_* + \lambda\|A\|_1\right) \quad subject\,to\ \ \|X - (N + A)\|_2 < \varepsilon$$

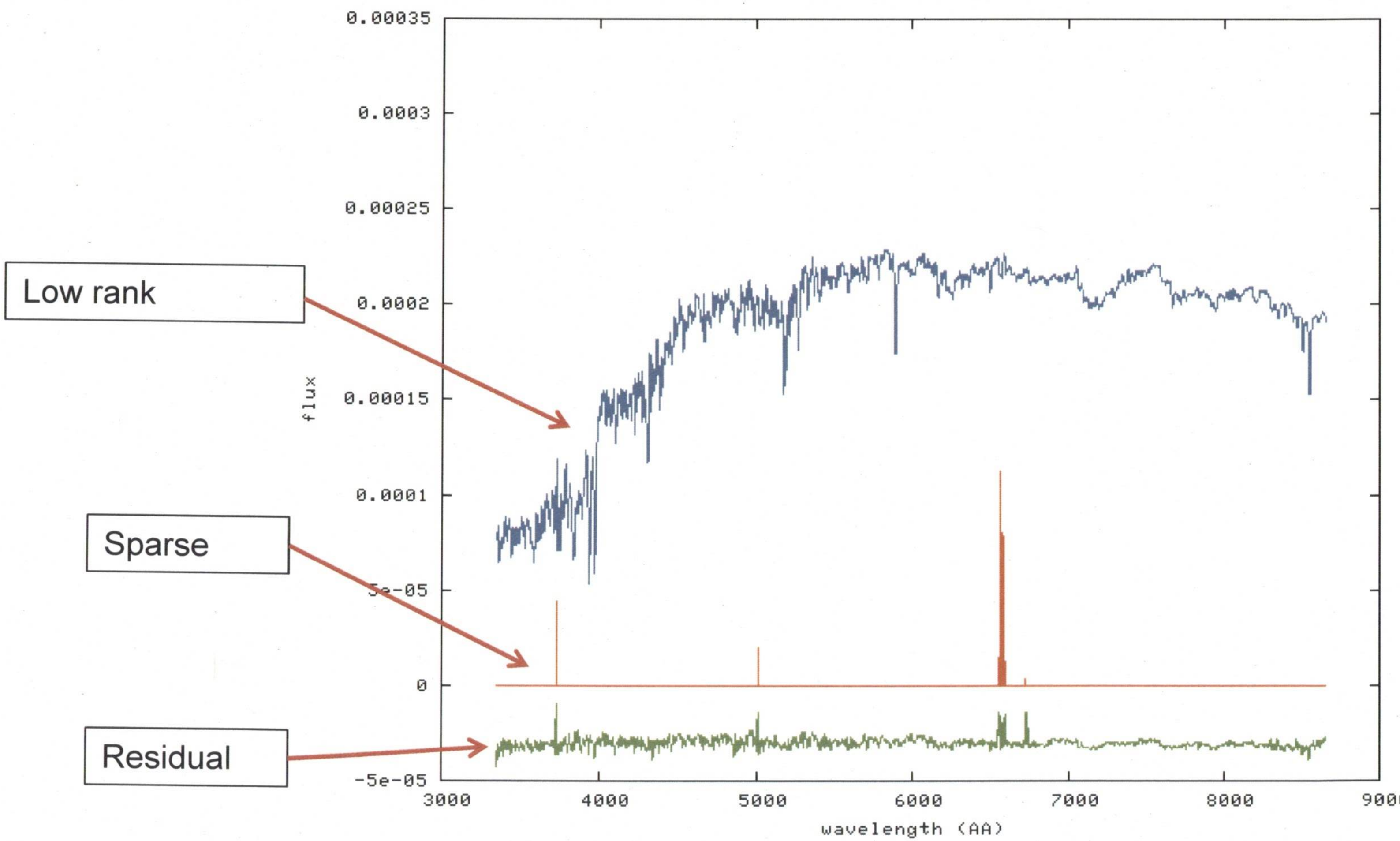
* E. Candes, et al. "Robust Principal Component Analysis". preprint, 2009.
Abdelkefi et al. ACM CoNEXT Workshop (traffic anomaly detection)

PCA



PCA reconstruction
Residual
0.00035
0.0003
0.00025
0.0002
0.00015
0.0001
5e-05
0
-5e-05
flux
3000
4000
5000
6000
7000
8000
9000
wavelength (AA)

Principal component pursuit



λ=0.6/sqrt(n), ε=0.03